Multi-scale Spatial Modeling for Logistics System Reliability Evaluations

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Abstract: This article proposes a multi-scale model suitable for describing patterns of store locations in a large-size supply-chain system. This model uses continuum approximation to simplify the detailed logistics network and supports various levels of decisions, such as robust logistics network designs and service area allocations for distribution centers. Several definitions and formulas are developed for evaluating reliability of logistics service and performance degradation of logistics networks when a distribution center does not perform to its full capability. Examples with store locations data taken from a large-size retail-chain illustrate impact of different network designs to system reliability and service degradation performance.

Acronyms

DC	distribution center
CA	continuum approximation
BPR	Bureau of Public Roads
MCMC	Monte Carlo Markov Chain

Notation

$\lambda(x, y)$	store location density
t_p	process time at DC
T_v	travel time from DC to stores
T_m	motion time for products

F_{T_v}	the travel time distribution
r	system reliability
p_i	the optimal re-routing proportion through DC_i
I_i	the relative importance of DC_i
ω_j	the normalized weights for difference components in global-scale model
μ_{xj}, μ_{yj}	the location for the j_{th} cluster centers
σ_j^2	the variance for the j_{th} cluster component

1 Introduction

Numerous studies examine problems in network reliability [1], system performance degradation and workload re-routing for telecommunication, power and transportation networks [17], [19]. In comparison, literature on modeling logistics system reliability or performance degradation is scarce. Logistics systems which involve moving goods, energy (e.g., electricity and gas), water, sewage, money or information from origins to destinations are critical to every nation's economic prosperity. Past studies [5] of road-network reliability addressed mainly connectivity and travel time reliability. These developments have limited use in providing first-cut analyses for system-level planning such as robust logistics network design to meet reliability requirements or supply-chain cost and delivery time evaluation for contract decisions [18]. Thus, the focus of this article is to develop an approximation model of large-size complex networks for supporting various types of system-level planning decisions.

System reliability in the logistics service sector is different from standard of reliability research of product designs. In particular, the impact of component degradation to system performance is rarely considered in product design, but is a critical factor in logistics systems. When the functionality of a DC or power plant is degraded, it may be necessary to re-route the needed material or electricity from other sources to maintain the system reliability. Reasons for logistics performance degradation include: insufficient supplies for demands at stores or DCs, capacity degradations at DCs, road conditions, weather situations or security inspection delays. This article uses several examples to illustrate the impact of network design to system reliability and performance degradation.

A typical large-size retail chain has more than 2000 stores in U.S. and numerous global- and regional-level distribution centers or transshipment points. The reliability of such a logistics system depends on the strategy of how the stores are supported by DCs, along with the capabilities of these DCs in moving materials in a timely fashion. When a particular DC's capability is not large enough to handle a given workload, a strategy for using other DCs to support it can change the reliability performance of the entire logistics system. Detailed logistics models are not equipped to handle such complications with a large and complex network. For system-level performance evaluations and planning decisions, continuum approximation (CA) analysis of logistics systems attempts to develop simplified approximations for potential system designs [8]. The CA approach first simplifies the logistics network by assuming that spatial locations (of customer, facilities, etc.) are typically modeled via a density function $\lambda(x, y)$ which describes the number of points per unit area as a function of position (x, y). Then, one can use this approximation model to develop resource allocation strategies. Section 4.1 shows that cluster-models used in epidemic studies [15] is useful in modeling the store locations.

This article proposes a multi-scale spatial approximation models for capturing key characteristics in logistics systems and applies a two-level model to illustrate logistics reliability evaluation procedures. Global-scale models are suitable for making decisions for large-scale operations that include global-level DCs with service areas across several states. County-level local models on the other hand characterize patterns on the number of stores in each county. Section 2 presents a simplified two-level logistics network. Section 3 defines logistics reliability and performance degradation measures, and describes optimal re-routing strategies when contingency occurs. Section 4 formally introduces our multi-scale modeling ideas. Numerical illustrations are provided in Section 5. Conclusion and future work are offered in Section 6.

2 Simplified Two-Level Logistics Network

The logistics systems in our partner companies are complicated with many DCs and thousands of stores and selling many types of products. For the purpose of illustration, this article considers a simplified logistics network with two levels: DCs and stores, and considers delivery time of products to stores, without regard to logistics costs. Each DC supports several stores in its surrounding area. In the case that a high impact contingent event partially or fully shuts down one DC (causing its capability degradation), items will be re-routed to support the stores not being served by the problematic DC. Note that the available DCs have their own demands to meet. With more products to handle, process time at DC could increase. Along with the potential increase of travel time due to re-routes, this creates a high chance of stock-outs at stores across the network.

Suppose there are M DCs (DC_1, DC_2, ..., DC_M) supplying a single kind of product for N ($N \gg M$) stores in an area A. Each DC_i will have its serving area A_i so that A_is are disjoint with $\bigcup A_i = A$. For simplicity, assume that there are sufficient products available in DCs (due to certain high-quality replenishment service from suppliers). Thus, we will not consider delivery time variations from suppliers to DCs. This implies that, the "motion time" (T_m), defined as time to move the products from origin to destination, consists two components: "process time" (t_p) at the DC and "travel time" (T_v) from DC to its stores. The process time includes loading, unloading, packaging, sorting time, waiting time for dispatch, and so on. Some traditional models consider the headway time (time between successive dispatches), which is absorbed into the variation of process time here to keep our model simple.

Process Time Model: Process time at a DC can be related to the total volume of products going through the DC; if volume increases, the process time per item may increase an amount depending on the limited capacity and manpower in the DC. The Bureau of Public Roads (BPR) traffic model [20] is used here to model process time at a DC as: $t_p(x, C) = t_a[1 + \beta(x/C)^\gamma]$, where t_a, $t_p(x, C)$, respectively, are free-flow process time and process time with flow x under the capacity C in the DC. The parameters β and γ provide for a nonlinear relationship between x and t_p. Denote by C_i the capacity of DC_i and let d_i equal the flow volume routing through DC_i. For now, assume that the traffic model parameters β and γ are the same for all DCs.

Given a sub-region A_i, the number of stores (N_i) in this sub-region would be given as $\iint_{A_i} \lambda(x, y)dxdy$, where $\lambda(x, y)$ is the store density defined over the whole region A. Similarly, given the demand density function as $\delta(x, y)$ per store, the expected total demand in A_i would be $d_i = \iint_{A_i} \lambda(x, y)\delta(x, y)dxdy$, which will be used as the flow volume routing through DC_i. See Section 4 for the assumptions and models of $\lambda(x, y)$. Note that in this presentation we assume that the demand does not depend on time t. To focus on the randomness from store locations only, we fix demand the same for all the stores , i.e., $\delta(x, y) = \delta$.

Travel Time Model: Travel time T_v is a ratio of distance (L) and a deterministic travel speed (v). For some simple forms of the distribution network, we can derive a closed form distribution of the travel distance L to the stores. For more complex settings, we may use simulations to approximate the travel distance distributions. Denote F_{T_v} as the distribution of T_v and F_L as the distribution of L.

Denote by L_i the random variable of the travel distance from the DC to stores in the sub-region A_i using direct shipping. We may assume that L_i have a log-normal distribution, $\log(L_i) \sim N(\mu_{L_i}, \sigma^2_{L_i})$.

In reality, there is more variability associated with travel time other than the distance along. We may use the multiplicative random noise to represent variation coming from speed, route condition, etc. For example, consider the model $T_{r,i} = (L_i/v) \times \epsilon$, where the noise random variable ϵ represents minor speed variations due to road and weather conditions. Assume the random variables $\log(L_i)$ and $\log(\epsilon)$ are independent, and $\log(\epsilon) \sim N(0, \sigma_\epsilon^2)$. Then, the distribution of travel time $\log(T_{r,i})$ is normal with the following mean and variance parameters: $\mu_{r,i} = \mu_{L_i} - \log(v)$, and $\sigma_{r,i}^2 = \sigma_{L_i}^2 + \sigma_\epsilon^2$.

3 Logistics Reliability and Service Degradation Models

3.1 Reliability Analysis for Single DC

Delivering products to stores according to schedule is important for meeting replenish requirements and preventing stockout (or over inventory). See [18] for more understanding of the importance of time factors in logistics operations. Define the service reliability of logistics networks, $r = Pr(T_m < t_0)$ as the probability of delivering products from DC to stores within a pre-specified limit t_0. Because $T_m = t_p + T_v$, $r = Pr(T_v < t_0 - t_p) = F_{T_v}(t_0 - t_p)$. Suppose that a special contingent event reduces proportion ξ of a DC's capacity. That is, in a given time period, the DC can only handle $(1 - \xi)$ of the products originally handled in its full capability. When $\xi = 1$, the DC is completely shut down. Service reliability will degrade as DC capacities degrade. The following two simple examples show how the service reliability depends on the store-location distributions.

Example 1: Consider a DC located in the center of a unit circle, where the store locations (x_i, y_i)'s are uniformly distributed within this circle. Denote $L_1 = (x^2 + y^2)^{1/2}$ to be the travel distance to visit the stores and denote F_{L_1} to be its distribution, $F_{L_1}(l) = P(L_1 < l) = P(x^2 + y^2 < l^2) = \pi l^2/\pi = l^2, 0 \le l \le 1$. Then, the corresponding probability density function is $f_{L_1}(l) = 2l$, $0 \le l \le 1$.

Consider the same setup for the DC, but the store locations (x_i, y_i)'s are from a bivariate normal distribution $f(x, y) = 2\pi\sigma^{-2} \exp\{-(x^2 + y^2)/(2\sigma^2)\}$. Then, the normalized squared distance $L_2^2/\sigma^2 = (x^2 + y^2)/\sigma^2$ has a χ^2 distribution with two degrees of freedom. See Figure 1 for the layout of the store distributions.

Set the original capacity $C_{original} = d$, $\sigma = 0.2$, $t_a = 1$, $v = 1$, and choose $\beta = 0.15, \gamma = 4$ according to common practice [20]. To ensure the service reliability equals to one ($r_{original} = 1$) in the original network, the time limit t_0 is set to $2 + \beta$. To keep the example simple, we neglect the noise factor ϵ in the travel time model. Thus, the distribution of travel time is the same as the

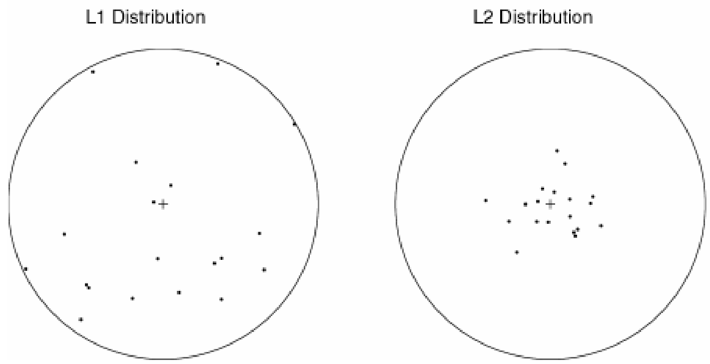

Figure 1: Layout of Store Distributions in Example 1

distribution of the distance (i.e., $F_{T_v} = F_L$).

When DC capacity degrades to C_ξ, the service reliability degrades correspondingly. The reliability of the two DCs described above are functions of the capacity: $r_i(C_\xi) = F_{T_{r,i}}(t_0 - t_p(C_\xi))$, $i = 1$ and 2. For the case with distance L_1, $r_1(C_\xi) = F_{L_1}(t_0 - t_p(C_\xi)) = [t_0 - t_p(C_\xi)]^2$, where $t_p(C_\xi) = 1 + \beta[C_{original}/C_\xi]^\gamma$. For the case with L_2, $r_2(C_\xi) = F_{L_2}(t_0 - t_p(C_\xi))$, where F_{L_2} is the cumulative distribution function of χ^2 with two degrees of freedom.

Figure 2 shows degradation of system reliability as a function of degradation in DC's capacity (i.e., $C_{original}$ becomes $C_\xi = \xi C_{original}$). Note that in the L_1 case, there is steady decrease of the reliability when the DC degrades and reaches zero for ξ around 0.87. In the L_2 case, the logistics system is more robust to capacity degradation and the reliability stays about the same for ξ between zero and 0.75. A quick drop of reliability occurs after capacity degrades over 0.75 and r_2 approaches zero as ξ increases to 0.87. The difference of the degradation patterns can be explained by the stores clustering effect in L_2 case.

3.2 Optimal Re-routing and System Reliability Analysis

This section considers the case of multiple DCs. Figure 3 uses an example to show the "service area" for each DC in a logistics network. Suppose a contingency shut down a DC, say DC$_k$. Then, the products originally routed through DC$_k$ will be re-routed through one or more of the other DCs. Note that these re-routes will place extra loads on the functioning DCs trying to maintain service levels. The additional motion time due to re-routes will impact the service reliability for the entire logistics network (and the cost of logistics operations from additional transportation and management costs).

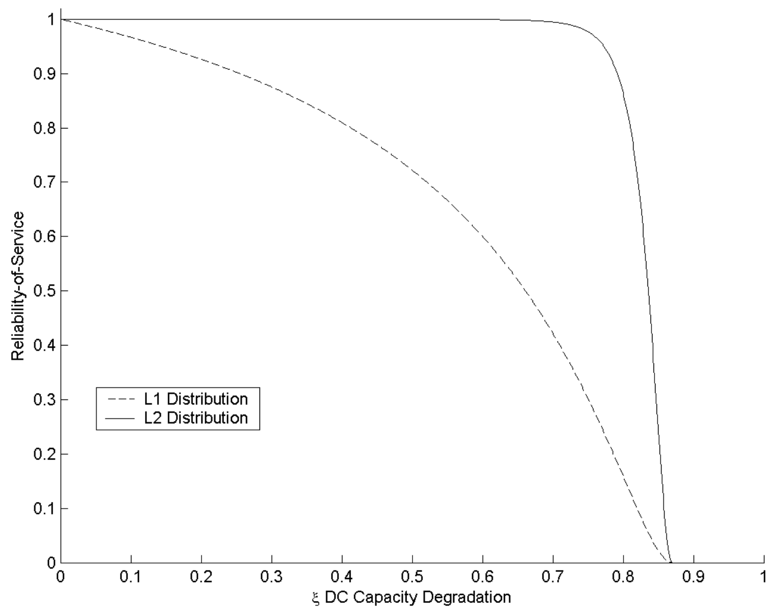

Figure 2: Service Reliability Degradation Patterns of Example 1

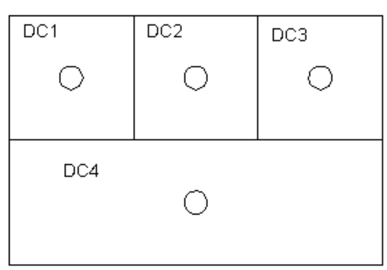

Figure 3: Layout of Logistics Network in Example 2

Our goal is to find optimal proportions assigned for re-routed products through the other DC. We also wish to investigate the relationship between DC$_k$'s performance degradation and the degradation of the system-level service reliability.

Re-routing Strategy: Suppose DC$_k$ (with demand d_k) is completely shut down upon a contingency. Let p_i = proportion of products re-routed from DC$_k$ to DC$_i$, where $i = 1, \ldots, k-1, k+1, \ldots, M$. For those demands $p_i d_k$ re-routing from DC$_i$, we assume that the travel distance increase by $l_{k,i}$, where $l_{k,i}$ is the distance between DC$_k$ and DC$_i$. That is, the re-routed products first arrive to where DC$_k$ is located, then the products are re-distributed using the same direct-shipping method.

Denote by $T^*_{r,k,i}$ the travel time for the amount $p_i d_k$ of products routed from DC$_k$ to DC$_i$ after contingency, then $T^*_{r,k,i} = l_{k,i}/v + T_{r,k}$. Denote the new process time for DC$_i$ as $t^*_{h,i}$, which can be calculated by using the new quantity d^*_i in place of d_i in the BPR model. Thus, for the products originally served by DC$_i$, the motion time is $T^*_{m,i} = t^*_{h,i} + T_{r,i}$. For those products re-routed from DC$_i$ to stores originally served by DC$_k$, the motion time becomes $T^*_{m,k,i} = t^*_{h,i} + T^*_{r,k,i} = t^*_{h,i} + l_{k,i}/v + T_{r,k}$.

When DC$_k$ is degraded by reducing proportion ξ of its capability, only $100\xi\%$ proportion of the demand d_k has to be re-routed. The procedure described above can be readily extended to this case by replacing d_k with ξd_k for evaluating degradation of system reliability.

System Reliability: When there are many DCs with different demands and reliabilities, a reasonable approach to describe the entire system's service reliability (r_{system}) is to take the weighted sum of the service probability in all sub-regions:

$$r_{system} = \sum_{i=1}^{M} d_i P(T_{m,i} < t_0) / \sum_{i=1}^{M} d_i$$

Based on the derivation of the distribution of the motion time ($T_{m,i}$) for each DC, the system-level reliability depends on the demand volume, DC capacities (affecting the process time) and network configurations (affecting the travel distances). When DC$_k$ is partially or completely shut down, this system-level reliability of service becomes

$$r^*_{system,k} = \frac{\sum_{i=1, i\neq k}^{M} d_i P(T^*_{m,i} < t_0) + \sum_{i=1, i\neq k}^{M} p_i d_k P(T^*_{m,k,i} < t_0)}{\sum_{i=1}^{M} d_i}. \tag{1}$$

The optimal re-routing strategy is defined to have the smallest decrease in r_{system} (or $r^*_{system,k}$). A DC is judged to be more important than another DC if its shut down causes a greater decrease in r_{system}. If the system is coherent, we expect r_{system} to decrease after the loss of one DC. Otherwise, it means that the original network is poorly designed.

DC_k	$r^*_{system,k}$	Optimal Re-routing Proportions	I_k
1	0.79	$p_2 = 0.26, p_3 = 0.45, p_4 = 0.29$	0.18
2	0.85	$p_1 = 0.17, p_3 = 0.59, p_4 = 0.24$	0.10
3	0.87	$p_1 = 0, p_2 = 0.5, p_4 = 0.5$	0.08
4	0.42	$p_1 = 0.11, p_2 = 0.13, p_3 = 0.76$	0.64

Table 1: Simulation Results of System Reliability after Contingencies in Example 3

A "*relative importance*" measure can be defined as the ratio of the reduction of r_{system} from DC_k to the reductions in r_{system} from all DCs (one at a time). That is, $I_k = \Delta r_{system,k} / \sum_{i=1}^{M} \Delta r_{system,i}$, where $\Delta r_{system,k} = r_{system} - r^*_{system,k}$. This ratio will provide us some guidelines allocating the budget to secure/maintain a particular DC's functionality.

The next example shows the optimal re-routing strategy in a hypothetic logistics network illustrated in Figure 3. See Section 5 for other examples based on real-life store locations.

Example 2: As illustrated in Figure 3, suppose there are four DCs located in a $300 \times 200\ km^2$ area A, with capacities 400, 400, 400 and 1000 items per unit time. Suppose there are a total of 600 stores in the area A, and the stores are distributed uniformly over the area. So that DC_1, DC_2, DC_3 will serve 100 stores each, and DC_4 will serve 300 stores. Let $(d_1, d_2, d_3, d_4) = (400, 400, 200, 1000)$ be the size of the expected demands for the four DCs per unit time. The parameters in the BRP traffic model are set as $\beta = 0.15$ and $\gamma = 4$. In addition, assume that $t_a = 2\ hours$, $t_0 = 6\ hours$, $v = 50\ km/hour$, and $\sigma^2_\epsilon = 1$. The relative distances between DCs: $l_{1,2}$, $l_{1,3}$, $l_{1,4}$, $l_{2,3}$, $l_{2,4}$,$l_{3,4}$ are 100, 200, 141, 100, 100, 141, respectively.

The original r_{system} is calculated as 0.93. Table 1 lists the system-level service reliability after a DC is shut down along with the optimal re-routing proportions. DC_4 is the most important among the four DCs for maintaining the system-level service reliability. Although DC_1 and DC_3 have equal capacities (and equivalent location positions), the drop in system reliability is almost 50% greater for DC_1 because it has more demand. Similarly, DC_1 and DC_2 have identical capacity and demand, but DC_2 is closer to the other DCs. Thus, the re-routing takes less effort.

By going through similar calculations with $100\xi\%$ of the DC capacity degraded, the system-level reliability degradation patterns are displayed in Figure 4. The results are analogous to Table 1. For example, shutting down a part of DC_4's capacity results in the sharpest drop for the system-level

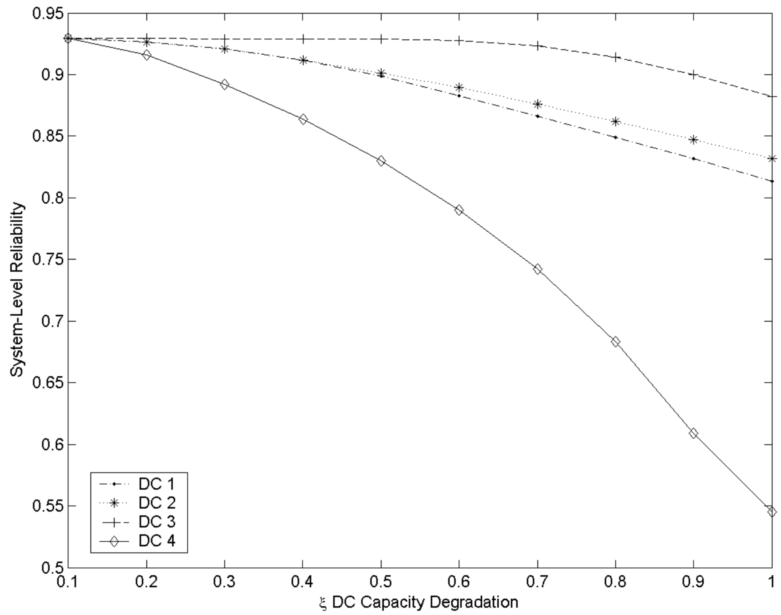

Figure 4: Degradation Path for the Four DCs in Example 2

reliability degradation due to its unique location. Because DC_1 and DC_2 are relatively close and have equal capacity, their degradation paths are similar.

The following present two technical observations in special cases.

Note 1. Suppose the capacities of all DCs are large enough so that their process time equals the free-flow time. That is, operators process products without any delay caused by resource constraints. Then, the re-routed work load will not cause any process time increase in the other DCs. Thus, the best re-routing strategy is to fulfill the demand of DC shut down via the nearest DC due to its least amount of increase in travel time.

Note 2: If DC_k shuts down, suppose the remaining DCs have the same demand, capacity, process time and travel distances to their respective stores. For each DC, the optimal proportion of re-routed products will be order inversely to its respective distance $(l_{ik}, i \neq k)$ to DC_k.

4 Multi-Scale Modeling of Store Distributions

In large-scale facility-location allocation problems, it's challenging to decide the number of DCs, their locations and allocation of stores by using the traditional deterministic optimization procedures [8]. The CA approach [7] simplifies the logistics network using an approximation model, and then develops allocation strategies based on this model. Multi-scale spatial data models provided in this article can provide various details of information to support decisions at different levels, including the strategic level, tactical level and operational level [4]. To keep our illustration brief, this section focuses on a spatial model at two levels: a large-scale model for a state, a small-scale model for individual counties. See Section 5 for numerical examples of utilizing these models in various logistics decisions concerning system reliability.

4.1 Large-Scale model of Store Distributions

For deciding the number and the location of global-level DCs along with the areas they served, a global model for store distribution density is needed to support a first-cut analyses of travel distances and customer demands. These analyses are important in optimizing strategic-level decisions of logistics network designs [18].

One of the key assumptions in many CA models is that discrete store locations arise from a homogeneous spatial Poisson process with a constant intensity [8]. This can be represented by $\lambda(x, y) = \rho/|T|$, where ρ is the constant rate of the process over the region T, and $|T|$ is the area of the region. This assumption is usually violated in practice because there will be many clusters around urban centers. Reilly *et al.* [15] apply mixtures of bivariate Gaussian densities to analyze clusters of point processes in an epidemiology study. See references in [14] and [10] for more examples. Besides clusters around metropolitan areas, there are a scattering of stores in rural areas. For this, we add a constant intensity term to the mixture densities. Specifically, the intensity has the following form:

$$\lambda(x, y) = \rho I[(x, y) \in T]\{\omega_0 \frac{1}{|T|} + \sum_{j=1}^{K} \frac{\omega_j}{2\pi\sigma_j^2} \exp\{-\frac{1}{2\sigma_j^2}[(x - \mu_{xj})^2 + (y - \mu_{yj})^2]\}\}, \tag{2}$$

where K is the number of the clusters, $I(\cdot)$ is an indicator function, $\{\mu_{xj}, \mu_{yj}\}_{j=1}^K$ represent the coordinates of cluster centers, $\{\sigma_j\}_{j=1}^K$ describe the spread of the clusters, $\{\omega_j\}_{j=0}^K$ are the normalized weights for different mixing components with $\sum_{j=0}^{K} \omega_j = 1$ and ρ measures the total density of points over the region T.

Bayesian methods have been applied successfully ([14], [13], [10]) to estimate parameters in the cluster analysis of epidemiology data. In our model, $\{\mu_{xj}, \mu_{yj}\}$ will be assigned conjugate Normal priors [16]. The prior distributions of σ_j^{-2} and ρ are independent Gamma, and the prior of $\boldsymbol{\omega}$ is a Dirichlet distribution [9]. The prior distribution of K is assumed to be either Poisson or discrete uniform distribution on $\{1, \ldots, k_{max}\}$ [16].

The likelihood for the m events $(x_i, y_i), i = 1, \ldots, m$, is given by the following commonly used *Janossy density*[6] in the point process model. Let $\boldsymbol{\theta}$ denote the model parameters:

$$L(\boldsymbol{\theta}) = \exp\{-\int_T \lambda(x, y)dxdy\} \prod_{i=1}^{m} \lambda(x_i, y_i). \tag{3}$$

Because metropolitan areas are small compared to states, we assume that the Gaussian probabilities are concentrated in a few local regions with limited spread. Thus, instead of integrating over the restricted region T (e.g., Texas), an approximation is used to integrate the normal densities over the domain $R^2 = (-\infty, \infty) \times (-\infty, \infty)$. Because integrations of individual normal densities in the mixture model are all equal to one, we have

$$\int_T \lambda(x, y)dxdy \approx \int_{R^2} \lambda(x, y)dxdy = \rho\{\omega_0 + \sum_{j=1}^{K} \omega_j\} = \rho.$$

In section 5.1, estimation results are used to validate this assumption. Using this approximation, the integration in (2) is avoided, and the likelihood becomes

$$L(\boldsymbol{\theta}) = \rho^m e^{-\rho} \prod_{i=1}^{m} \{\omega_0 \frac{1}{|T|} + \sum_{j=1}^{K} \frac{\omega_j}{2\pi\sigma_j^2} \exp\{-\frac{1}{2\sigma_j^2}[(x - \mu_{xj})^2 + (y - \mu_{yj})^2]\}\}. \tag{4}$$

Monte Carlo Markov Chain (MCMC) methods ([13], [11]) can be implemented to compute or generate samples of the joint posterior distribution of model parameters. For brevity, the MCMC details are not discussed here. See Section 5.1 for numerical illustrations.

4.2 County Level Model for Store Counts

For logistics planning at the county-level a more detailed model is needed. Here, we focus on modeling strategies of store counts aggregated within a county by considering the local covariate information and the effects of surrounding areas. Using a more detailed and accurate model of store counts, one can make small-scale decisions. For example, when the prediction of store counts over a neighborhood of counties exceeds over a given threshold, a local DC can be established to serve these stores. The county level model is also useful in calculating optimal vehicle routing lengths to serve stores in a county such that transportation costs can be derived for planning regional level logistics activities [12].

Suppose that the study region consists of N_c subregions (e.g., counties) a_i, $i = 1, \ldots, N_c$, and $\{n_i\}_{i=1}^{N_c}$ are the number of stores within each subregion. These count data are modeled with a spatially correlated Poisson distribution as $n_i \sim Poisson(\lambda_i)$, where λ_i represents the expected store counts in county i. The following shows the flexibility of modeling the λ_i's:

$$log(\lambda_i) = log(\int_{a_i} \lambda(x,y)dxdy) + g({\boldsymbol z_i}^T \boldsymbol\beta) + u_i + v_i. \quad (5)$$

The first term integrates the intensity function (2) over the county region a_i to create a "*linkage*" between the large-scale and small-scale models through their means. The Poisson mean $log(\lambda_i)$ is also affected by local covariates $\boldsymbol z_i$. The last two terms are the small-scale random effects representing the variations not captured by the two mean terms. Specifically, the independent errors are modeled by normal variates $(v_1, \ldots, v_{N_c})$'s with mean zero and variance σ_v^2, and the spatially dependent random effects are modeled by $(u_1, \ldots, u_{N_c})$'s. To keep the modeling of the spatial effects simple, we adopt the following model developed by Besag *et al.*[2] for the distribution of relative risks of a disease. In this model, u_i's have conditional autoregressive gaussian (CAR) distribution,

$$\begin{aligned} u_i | u_{j,j\neq i} \quad &\sim \quad N\left(\sum_{j\neq i} w_{ij}u_j/W, \sigma_u^2/W\right), i = 1, \ldots, N_c \\ \text{and} \qquad & W = \sum_{j\neq i} w_{ij}, \end{aligned}$$

where w_{ij}s denote weights defining which regions j are neighbors to region i ($w_{ii} = 0$ by convention). Typical applications consider $w_{ij} = 1$ if region j is adjacent to region i, $w_{ij} = 0$ otherwise.

In the parameter estimation process, the parameters in the first term of (5) are set to equal the estimates from the large-scale modeling stage for the needed "linkage" between two different levels of models. The priors of $\boldsymbol\beta$ can be non-informative as applied in objective Bayesian procedures [3]. The parameters σ_u and σ_v are independent Inverse-Gamma random variables.

The likelihood of the store counts is an integral of the product of N_c independent Poisson probability mass functions multiplied by the density of the random-effects λ_i's as described in (5) (in a log-transformed scale). The WinBug MCMC procedure [21] is implemented to obtain the samples of posterior distributions of model parameters. See Section 5.1 for an example of numerical illustrations.

5 Applications

5.1 Multi-level Modeling Results for Store Locations

The store locations data for our illustrations were contributed from a major retailer. Like other states, Texas has metropolitan areas (e.g., Dallas, Houston, San Antonio and Austin) represented by

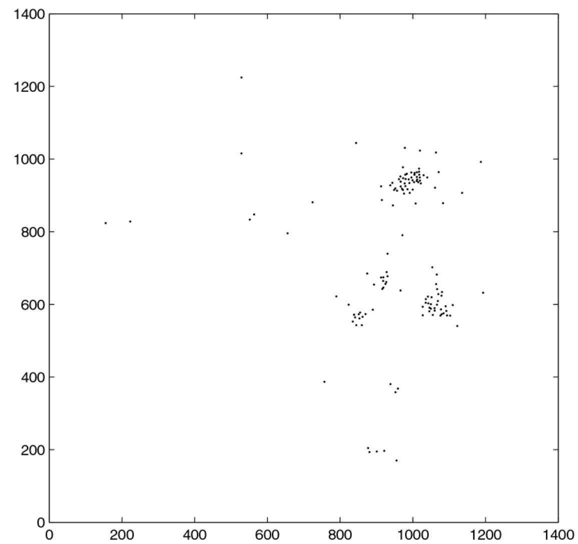

Figure 5: Distribution of the stores

dense store clusters. This section applies the multi-level modeling procedures described in Section 4 to the 142 stores in the 254 counties of Texas. See Figure 5 for the graphical distribution of these stores.

In large-scale modeling, the number of clusters is estimated as four and the center of the clusters are at the central location of the four largest cities in Texas. Table 1 summarizes the posterior means of the weights ω and spread σ^2 based on 50,000 runs of MCMC iterations after a burn-in of 5,000 iterations. Figure 6 displays the posterior mode of the cluster assignments and Figure 7 shows the contour of the intensity function, where the outer rings of the circles (representing the clusters) are drawn at the three-sigma line of the bivariate normal density. Although there are some stores outside the three-sigma areas, the mixture of clusters models stores locations around the four largest cities reasonably well. The stores not captured by these normal mixtures are modeled from the homogeneous Poisson distribution with the constant intensity estimated as $\hat{\rho}\,\hat{\omega_0}/|T|$, where $\hat{\rho} = 140$, $\hat{\omega_0} = 0.27$ are the posterior means of ρ and ω_0.

A 80×90 grid is used on the Texas map (Figure 8) to approximate the integration of the estimated Poisson intensity over the irregular shape of those counties. This integration based on the large-scale modeling results provides the first component of the $log(\lambda_i)$ in (5) for the small-scale model. We choose the county population estimates data from the U.S. Census Bureau as the covariates z, and

cluster	0	1	2	3	4
ω	0.27	0.08	0.08	0.21	0.36
σ	-	10.4	12.3	21.6	25.3

Table 2: Posterior Means of Parameters in the Large-scale Model

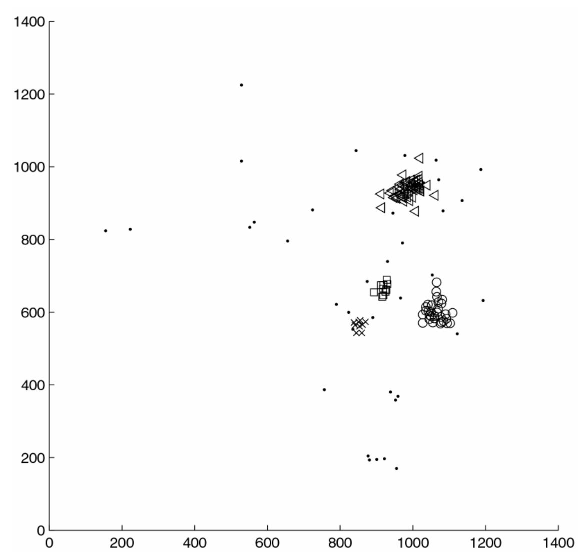

Figure 6: Posterior Mode of the Cluster Assignments

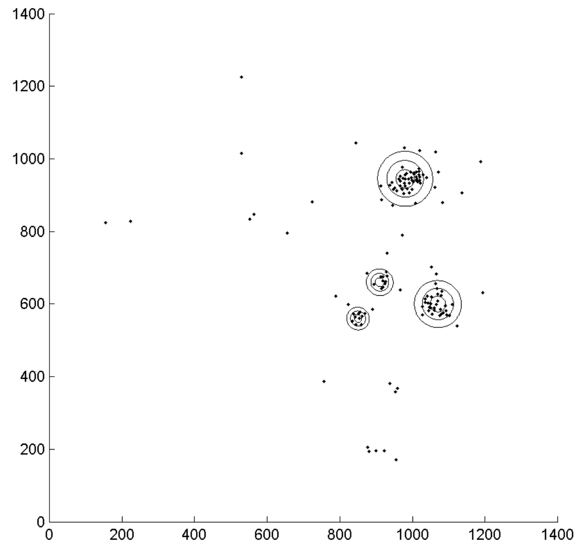

Figure 7: Intensity of the Inhomogeneous Poisson Process

assume $g(z^T\beta)$ to be have a linear form as $z^T\beta$.

The Bayesian estimation procedure is implemented in Winbug [21]. After a burn-in of 10000 iterations, the posterior means (and standard deviations) for σ_u and σ_v are calculated as 0.154 (0.164), 0.065 (0.054), respectively, based on 50,000 iterations. The posterior mean and standard deviation of β are -5.48×10^{-4} and 0.0063, after we scale the covariates z_is to proper range as z_i =county i's population /100000. The county population doesn't provide much information in our small-scale model of the store counts. Other choices of covariate $\boldsymbol{z}$ and forms of $g(\boldsymbol{z}^T\boldsymbol{\beta})$ will be examined in future studies. See Figure 8 for the estimated store counts from the models.

5.2 Optimal Re-routing Strategy and Logistics System Reliability

This section contains examples to illustrate the optimal re-routing strategy after contingency based on service reliability measures of logistics networks. Future work described in Section 6 will consider other measures such as cost and system-robustness (to handle unexpected events) in these decisions.

Example 3. Suppose local DC assignment is based on the number of stores in a county exceeding a fixed threshold (e.g., $\lambda_0 = 10$ stores). The determination of an optimal threshold value might be computed using sensitivity analysis. Based on the estimation provided by our model, there are five among 254 counties with more than 10 stores. Figure 9 presents the locations of these five

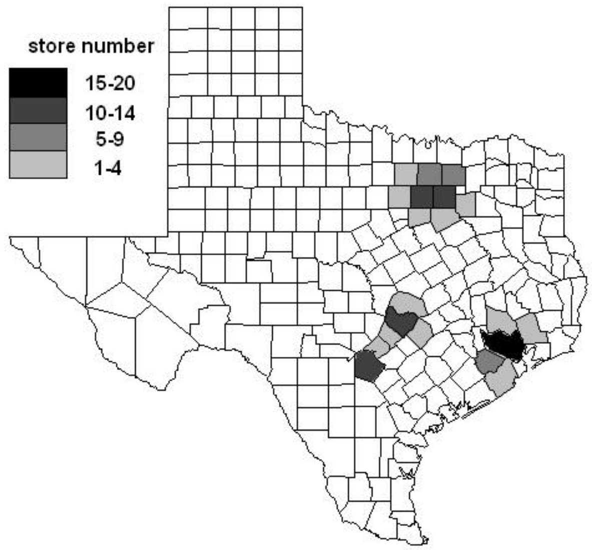

Figure 8: Estimated Store Counts data for Counties in Texas

DCs. From these locations, the relative distances (in miles) of DCs is calculated in the following matrix:

$$\mathbf{L} = \begin{pmatrix} 0 & 40 & 283.59 & 394.24 & 358.9 \\ 40 & 0 & 292.62 & 406.72 & 348.2 \\ 283.59 & 292.62 & 0 & 116.62 & 170.88 \\ 394.24 & 406.72 & 116.62 & 0 & 223.61 \\ 358.9 & 348.2 & 170.88 & 223.61 & 0 \end{pmatrix}$$

Determination of the DC service areas is a on-going research work [18], so for the purpose of illustration, the simple area allocations used here are illustrated in Figure 9.

Suppose every store has the same demand (d_0) during the unit time studied. The total demand in the area A_i ($i = 1, 2, \ldots, 5$) is $d_i = \iint_{A_i} \hat{\lambda}(x, y) d_0 \; dxdy$ based on the intensity $\hat{\lambda}(x, y)$ estimated from the large-scale model. The total demand per unit time for the five areas are: $D_i = (31, 31, 23, 20, 37)d_0$. For the system reliability analysis, the capacity (C_i) is set to the demand of the regional DCs so that extra capacity is not an issue in this example. The parameters of the logistics models describing the process and travel time are set as $\beta = 0.15$, $\gamma = 4$, $t_a = 4\ hours$, $t_0 = 12\ hours$, $v = 50\ km/hour$ and $\sigma_\epsilon^2 = 1$ based on results given in the literature [20] and direct feedback from practitioners.

Following the procedure in Section 2, the network reliability presented in Figure 9 is $r_{system} = 0.914$. The optimal re-routing proportions are listed in Table 3 for the five scenarios that result from

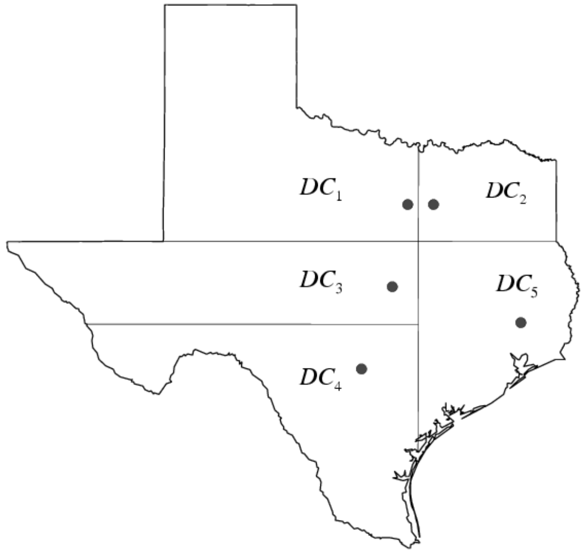

Figure 9: DC Locations and Service Regions for Example 3

DC_k	$r^*_{system,k}$	Optimal Re-routing Proportions	I_k
1	0.86	$p_2 = 0.75, p_3 = 0.18, p_4 = 0, p_5 = 0.07$	0.16
2	0.86	$p_1 = 0.76, p_3 = 0.17, p_4 = 0, p_5 = 0.07$	0.16
3	0.88	$p_1 = 0, p_2 = 0.26, p_4 = 0.16, p_5 = 0.58$	0.10
4	0.88	$p_1 = p_2 = 0, p_3 = 0.28, p_5 = 0.72$	0.10
5	0.76	$p_1 = 0; p_2 = 0.45; p_3 = 0.33; p_4 = 0.22$	0.48

Table 3: Example 3's System Reliability After Shutting Down One DC

a single DC shut down.

In this example DC_1 and DC_2 act as a parallel system; if one DC fails, the products can be re-routed through the other DC in a short time. This makes the system reasonably robust; when one of DC_1 or DC_2 fails, the drop of system reliability is only about $5\% = (91\% - 86\%)/91\%$. A similar observation can be made for DC_3 and DC_4, where the decrease of system reliability is only 3%. DC_5 has a relatively larger demand and longer distance to other DCs. Thus, when DC_5 shuts down, the system reliability is dropped more (15%). Its relative importance measure $I_5 = 48\%$ is at least three times larger than others.

Example 4. From the last example, suppose DC_1 and DC_2 are merged with a single distri-

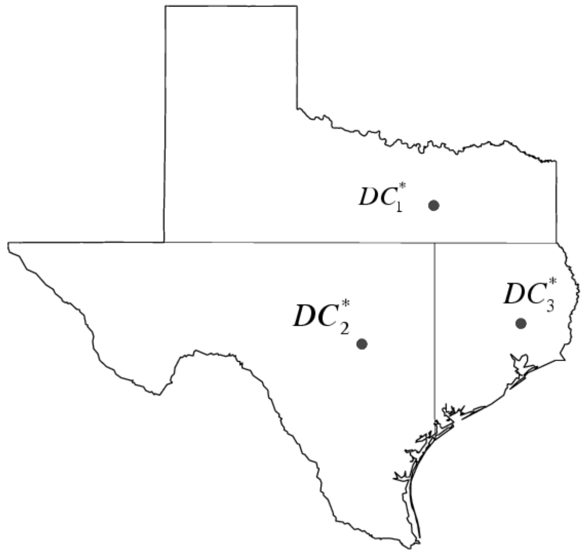

Figure 10: DC Locations and Service Regions for Example 4

DC_k^*	$r^*_{system,k}$	Optimal Re-routing Proportions	I_k
1	0.41	$p_2 = 0.52, p_3 = 0.48$	0.54
2	0.71	$p_1 = 0.63, p_3 = 0.37$	0.22
3	0.69	$p_1 = 0.51, p_2 = 0.49$	0.24

Table 4: Example 4's System Reliability After Shutting Down One DC

bution center DC_1^*. Also suppose DC_3 and DC_4 are merged into DC_2^*, while DC_5 (now denoted DC_3^* remains the same (see Figure 10).

The relative distances between DCs are: $\text{DC}_1^* - \text{DC}_2^* = 349.6$, $\text{DC}_1^* - \text{DC}_3^* = 356.5$ and $\text{DC}_2^* - \text{DC}_3^* = 190.3$. The total demand per unit time for the three areas are then calculated as $D_i^* = (62, 43, 37)d_0$. The other parameters are fixed at the values set in Example 3.

The reliability of this new network is $r_{system} = 0.915$. The system reliability after one DC shuts down and the optimal re-routing proportions are summarized in Table 4. Although the original system reliability is almost the same as the reliability obtained in Example 3, there is a significant reduction in reliability upon DC failure, especially DC_1^*. Obviously, this system is not as robust as the system presented in Example 3, and this loss of reliability offsets some or all of the cost benefits gained by aggregating DCs.

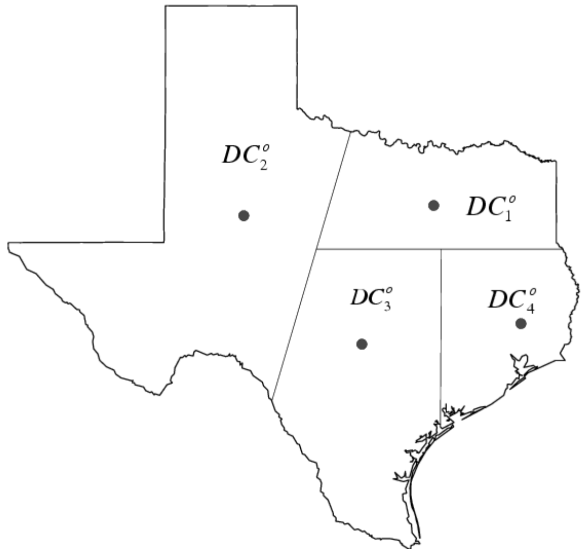

Figure 11: DC Locations and Service Regions for Example 5

Example 5. Both examples given above placed DCs at metropolitan areas. This example examines the effect of placing an additional DC (denoted as DC_2^o) in a less urban area as shown in Figure 11. The system and model parameters are set at the same values as Example 4. However, the demands (and capacities) are recalculated correspondingly as $D_i^o = (52.5, 21.5, 31, 37)d_0$, and the relative distances of the new DC to the original three DCs in Example 4 are: $\mathrm{DC}_1^o - \mathrm{DC}_2^o = 295.7$, $\mathrm{DC}_2^o - \mathrm{DC}_3^o = 300.0$, and $\mathrm{DC}_2^o - \mathrm{DC}_4^o = 446.5$.

The reliability of this new network is $r_{system} = 0.952$, slightly higher than the previous examples. Table 5 presents the re-routing proportions and their corresponding reliabilities. Note that the reductions in system reliability are less than those in Example 4, but more than in Example 3. This shows how the redundancy at the clustered metropolitan areas (in Example 3) made the system relatively more robust with regard to DC shut down. In this case, the relative importance of DC_1^o is much larger than the others due to its heavy demand.

Example 6. This logistics network illustrated in Figure 12 replaces DC_1 and DC_2, by $\mathrm{DC}_1^{'}$ and keep other 3 DCs at the same position as given in Example 3. This example compares the logistics-network reliability with the four-DC case in Example 5, and investigates the degradation of logistics-network reliability. The demands (and capacities) are $D_i^{'} = (62, 23, 20, 37)d_0$. The relative distances of the DCs are: $\mathrm{DC}_1^{'} - \mathrm{DC}_2^{'} = 290$, $\mathrm{DC}_1^{'} - \mathrm{DC}_3^{'} = 400.5$, and $\mathrm{DC}_1^{'} - \mathrm{DC}_4^{'} = 356.5$.

The original reliability of this network is $r_{system} = 0.920$. Table 6 presents the re-routing pro-

DC_k^o	$r^*_{system,k}$	Optimal Re-routing Proportions	I_k
1	0.53	$p_2 = 0.09, p_3 = 0.45, p_4 = 0.46$	0.5
2	0.84	$p_1 = 0.42, p_3 = 0.12, p_4 = 0.46$	0.13
3	0.81	$p_1 = 0.50, p_2 = 0.03, p_4 = 0.47$	0.17
4	0.78	$p_1 = 0.62, p_2 = 0, p_3 = 0.38$	0.20

Table 5: Example 5's System Reliability After Shutting Down One DC

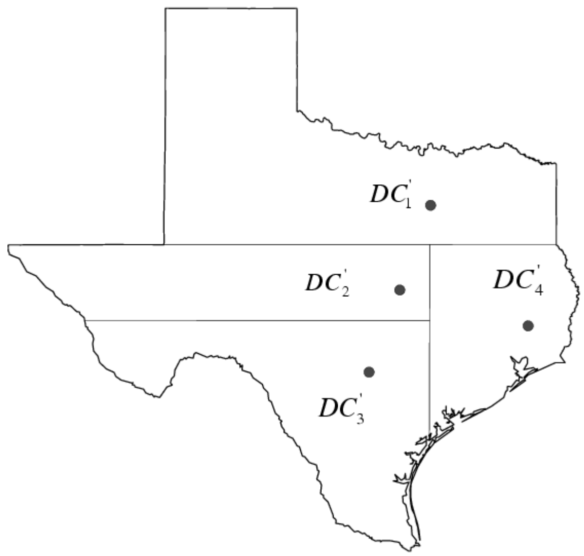

Figure 12: DC Locations and Service Regions for Example 6

portions and their corresponding reliability. Because the concentrated demand in $DC_1^{'}$, its relative importance is the highest. Due to their low demands, $DC_2^{'}$ and $DC_3^{'}$ have smaller relative importance than the other DCs.

Figures 13 and 14 graph the reliability degradation paths for Examples 5 and 6 after partially shutting down one DC. The degradation paths for DC_1^o and $DC_1^{'}$ have similar nonlinear degradation patterns. For $DC_2^{'}$ and $DC_3^{'}$, the reliability decreases slightly as the DCs degrade. In example 5, on the other hand, the partial shut down of DC_2^o and DC_3^o cause greater loss in system reliability. Due to having same location and demand, the effect of partial shut down of DC_4 are similar for both examples.

$\mathrm{DC}_k^{'}$	$r^*_{system,k}$	Optimal Re-routing Proportions	I_k
1	0.43	$p_2 = 0.30, p_3 = 0.22, p_4 = 0.48$	0.66
2	0.88	$p_1 = 0.31, p_3 = 0.16, p_4 = 0.53$	0.06
3	0.88	$p_1 = 0, p_2 = 0.28, p_4 = 0.72$	0.06
4	0.75	$p_1 = 0.44, p_2 = 0.33, p_3 = 0.23$	0.22

Table 6: Example 6's System Reliability After Shutting Down One DC

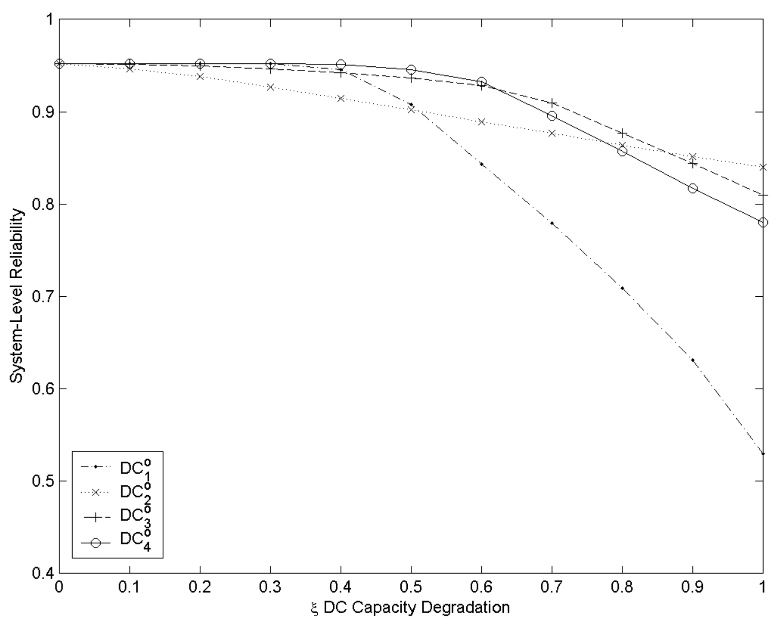

Figure 13: Degradation Path for DCs in Example 5

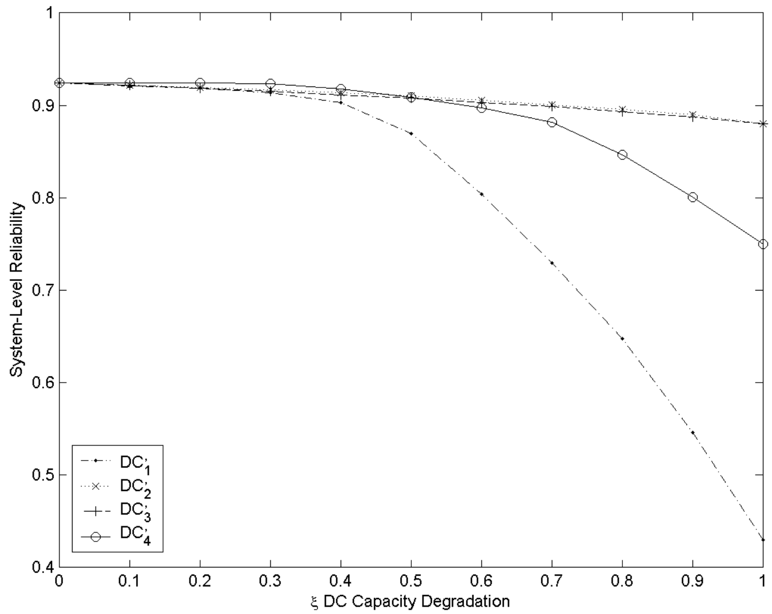

Figure 14: Degradation Path for DCs in Example 6

6 Conclusion and Future Work

This article develops a multi-scale spatial model for characterizing patterns of store locations in a large-size supply chain system. The approximation models greatly simplify the detailed logistics networks. The different levels of our model support various levels of decisions in logistics planning. This paper introduces reliability measures for the logistics networks and develops a simple optimal re-routing strategy to handle possible contingencies. The service degradation patterns are illustrated by several examples to examine their relationship with capacity degradations in DCs.

The optimal re-routing strategy is developed for the case with only one DC shut down. When there are more than one DCs shut down, optimizing re-routing strategy needs further work. Moreover, when we consider the transportation costs and other costs of DC operations, the optimal locations of DCs and the re-routing strategy would be more complicated. How to design a system with minimal cost and robust against possible capability degradations (or fully shut down) in various number of DCs is a challenging problem. In addition, if the connectivity between DCs and stores is not fully guaranteed, the system reliability evaluation and robust system design issues become even more interesting. This article shows the potential of research in the logistics-network reliability and design areas.

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